

Sudoku Hints

Long Version Help for Your Healthy Addiction!

I know you've seen it in your paper, you may even have played it, but have you truly mastered the art of Sudoku? For me, Sudoku is not just a puzzle, it's an addiction. The purpose of this article is to give you some tips for mastering Sudoku.

A Sudoku puzzle is comprised of a 9 x 9 grid of cells broken down further into 9 squares. The squares are comprised of a three by three array of cells within the larger and are usually delineated by shading or darker borders. (See Figure 1) The goal of the puzzle is to place the digits 1 through 9 throughout the puzzle such that every row, column and square contains all the digits 1 through 9. The puzzle creator fills in a small number of the cells as clues to the final configuration. In general, the fewer cells provided, the more difficult the puzzle is to solve. Your job is to use those clues to fill in the rest.

1					8			7
4		5	3			6		
			7	4	5	3		
					3		7	2
				6				
7	3		1					
		4	9	3	6			
		6			4	9		3
3			5					6

Figure 1. Example Sudoku puzzle

Because of the requirement to contain the values 1-9 in each row, column, and square, each time a cell's value is determined it will impact those three dimensions. In other words, each time you establish the value of a cell you implicitly forbid that value anywhere else in that row, column and square. We will use this property to help determine the rest of the values.

The methodology described below I call the "punch out" method. In a nutshell what it does is to write every remaining possible value of each cell within that cell. These are determined by looking at the row, column and square of each cell to "punch out" the invalid values. In theory a completely empty puzzle would have each cell containing all values one through nine. Each clue provided by the puzzle creator and each cell that you complete, punch out potential values along each axis until only one possible value is left for a given cell.

Step 1 - Look around the cells for obvious solutions. Sometimes there are enough clues given for a value that you can solve some cells.

1					8			7
4		5	3			6		
			7	4	5	3		

Figure 2. Example of obvious cells

In the above figure, there is a 7 in the second and third square but not in the first. The seven in row one and row three mean that the first square's seven must be in row two. Since there's only one opening in row two, the 7 must be in the second column. You can almost always find some values to fill in. I proceed through the puzzle from the top to the bottom looking at each of the values 1 through 9 individually, looking for clues such as this.

If you don't find many or even any, don't worry as this first step merely simplifies subsequent steps but the answer will still fall out using this method.

Step 2 - Work through the puzzle filling in the possible values for each cell. There are many ways to accomplish this, I'll introduce two:

1. Cell-by-cell - for each cell look along it's row, column and in it's square for each value 1-9. If the value isn't already taken then it's a possible for the cell and you annotate it. Continue through all numbers and all cells.
2. For-each-number - Work each number cell by cell. Start with 1 and for each square that doesn't have a 1 in it, determine the cells within it that could contain a 1 and annotate a 1 in there. Continue for each value 2-9.

Once this is complete you will have all the possible values annotated for each "empty" cell. Please note that it is very important to be careful when filling in these values as any mistake here will result in an invalid solution.

Using Figure 1 above, after filling in all possibilities you would now have the following:

1	269	3	26	269	8	245	2459	7
4	7	5	3	129	129	6	1289	189
2689	12689	1289	7	4	5	3	1289	189
5689	145689	189	48	589	3	14589	7	2
2589	124589	1289	248	6	7	14589	3	14589
7	3	289	1	2589	29	458	45689	4589
2589	1258	4	9	3	6	7	1258	158
258	1258	6	28	1278	4	9	1258	3
3	1289	7	5	1278	12	1248	1248	6

Figure 3. Sudoku puzzle with possible values filled in.

The easily found values are in red and the possibilities are in smaller font. Note that all the threes and sevens were able to be placed in the puzzle.

Step 3 - Use clues within the puzzle to reduce the possibilities for each cell until you can fill in all the vacancies. Since there exists a relationship between cells along rows, columns and within squares, every time you enter a new cell value you have to punch out that value as a possible along each axis from all other cells in the same row, column and square.

- A. Clue 1 - One Remaining Possible - This occurs when all the values but one for a cell are punched out by other cells in the same row, column or square. This is the most obvious solution and is ultimately the goal of the puzzle. The sole remaining possible is your solution for that cell.
- B. Clue 2 - Only Possible - a related solution technique is to review all numbers 1-9 in each row, column & square looking for a cell that is the only cell in that row, column or square containing that value as a possibility. Since each row, column and square must contain 1 occurrence of each value this cell must be it. In Figure 4, the row is missing values of 4, 5 & 9. Only 1 cell (column 2) has 9 as a possible solution, so that cell must be the 9 even though that cell contains other possibilities. Check each row, column and square looking for these situations.

45	59	6	7	2	45	1	3	8
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Figure 4. Example of Only Possible in Row.

- C. Clue 3 - Twins - when two cells in a row, column or square have exactly the same two remaining possibilities than you have twins. No matter which value is right for one the other value must be correct for the twin cell. This situation enables you to clear out additional possible values along the shared axis. If the twins are in the same square, then you can remove the twins from all other cells in the square. If the twins share a row or column, whether or not they share a square, then you can punch out both values as possibilities along that shared axis.

247	167	12567	9	45	45	8	124567	3
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Figure 5. Example of Twins

In the above example, the 5th and 6th cells are twins. They contain the exact same possibilities. Since they share a row and square you can remove both 4 and 5 from all cells in the row and square that they share. This also applies to triplets though they occur far less often.

- D. Clue 4 - Pointers - A clue related to twins occurs when a pair of aligned cells in a square contain the only instances of a possible value for the square. While we don't know which will have the value and therefore can't solve their cells, we do know that one of them will have that value thereby covering that value for the shared row or column. When this occurs you can use the alignment as a pointer to remove that possible from all other cells along the pointed row or column. In short, they point to values that can be removed.

						3	8	7
47	8	14567	145	9	3	146	146	2
						5	16	9

Figure 6. Example of Pointer.

In the above figure, cells 7 & 8 of the 2nd row contain the only possible value of 4 for their square. Since they line up along a row they point to other cells from which you remove 4 as a possible. Punching out 4 from the first cell solves it using Only Remaining. You can't say anything about any of the columns because they don't point that way.

- E. Clue 5 - Reverse Pointers - A related situation occurs when only 1 square has a possible value for a given row or column allowing you to eliminate that value as a possible for the other cells within the square. Rather than allowing us to solve a cell this reduces the number of possibilities for 1 or more cells. I refer to this as a reverse pointer because they don't point out to a row or column but rather back into their square.

1	269	3	26	269	8	245	2459	7
4	7	5	3	129	129	6	1289	189
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5689	145689	189	48	589	3	14589	7	2
2589	124589	1289	248	6	7	14589	3	14589
7	3	289	1	2589	29	4 58	4 5689	4 589
2589	1258	4	9	3	6	7	1258	158
258	1258	6	28	1278	4	9	1258	3
3	1289	7	5	1278	12	1248	1248	6

Figure 7. Example Reverse pointer.

In row 6 of the above example, neither the left square nor the center square for this row has 4 as a possible value, that means the 4 must be in the 6th row of the right most square. Since the square can only have one 4, this allows you to remove the other 4's in that square that aren't in row 6. We know one of the row 6 cells will be the 4 but not yet which. Remember the goal is to reduce the possibilities for each cell until you can solve them so this may reduce the number of possibilities allowing another clue to solve.

While filling in values and removing possibilities be on the look out for conflicts. Conflicts occur when two cells in a row, column or square are down to 1 possible and it's the same value or when a cell has no remaining possibilities. These situations indicate that you've made a mistake somewhere and you will likely have to restart.

The clues given above are sufficient to solve nearly all puzzles if you are very careful and systematic in following them. Once in a while I've been forced to guess in the very hardest puzzles. In these rare cases, I circle all values I select from that point on so that I can reset the puzzle if I find a conflict and have to guess another value.

Good luck and enjoy the challenge!

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